

Fall Quarter 2010

UCSB Physics 225A & UCSD Physics 214

Homework 1

Problem 2 has nothing to do with what we have done in class. It introduces somewhat strange coordinates called “rapidity” and “pseudorapidity” that are used in hadron colliders. By working through this problem hopefully you will understand what these are and why they are used.

Problem 3 is a bit of a (fun?) tour de force in differential equations etc. It may not be worth spending too much time working out the details, but, especially if you are an experimenter, pay attention to the main conclusion, which is that the spatial deviation and the angular deviations are correlated and the correlation is calculable.

Problems 4 and 5 should be reasonably straight forward.

• Problem 1

These are some introductory questions which will introduce you to the Particle Data Book and will help you to get some feeling for the important scales of particle physics.

(a) Look up the masses of the W and Z bosons. Compare them with the proton mass. Estimate the approximate range of the forces mediated by the W and Z . Compare with the size of the proton.

(b) Look up the lifetimes of the μ^+ , τ^+ , K^+ , π^+ , π^0 , ρ^0 , D^+ , B^+ , J/Ψ . From the values of the lifetime, what are the interactions responsible for these decays? Calculate the average distance travelled in the LAB by all of these particles when they are produced with a momentum of 10 GeV/c.

Note, for your information: the quark contents of the hadrons in this question are the following. $K^+ = \bar{s}d$, $\pi^+ = u\bar{d}$, $\pi^0 = (\bar{u}u - \bar{d}d)/\sqrt{2}$, ρ^0 same as π^0 , $D^+ = c\bar{d}$, $B^+ = \bar{b}u$, and $J/\Psi = c\bar{c}$. These are all $q\bar{q}$ states, aka *mesons*. Except for the ρ^0 and the J/Ψ , all the mesons in this question are the lowest lying states for a given $q_i\bar{q}_j$ combination. These low lying states have total internal angular momentum (orbital + spin) for the $q_i\bar{q}_j$ pair equal zero. This internal angular momentum can be identified with the spin of the meson. The ρ^0 and $J\Psi$, on the other hand, have spin 1.

- (c) Look at the possible decay modes of the muon and the tau. Why are the decay modes of these two charged leptons so different ?
- (d) At the top of the atmosphere, high energy cosmic rays produce an equal number of π^+ and π^- which then decay. Their decay products can then decay further. What is the ratio of the number of electron neutrinos to muon neutrinos that reach the earth. Assume that all unstable particles decay before hitting the earth, which is not a correct assumption.
- (e) An experimentalist has built a 3 cm x 3 cm silicon detector to measure accurately the trajectories of charged particle. This detector has many readout channels with a spacing of 50 microns. To check that the detector works, in the past he has travelled to an accelerator, set up his detector in-between two known working detectors, and let a beam of particle pass through his apparatus. He is now tired of traveling, and he decides to use cosmic rays at sea level instead. How long will it take him to collect enough data so that on average at least 1000 cosmic ray muons will pass through each readout channel.
- (f) Look at what is known about the branching ratios for $K^+ \rightarrow \pi^+\pi^-\pi^+\nu_e$ and $K^+ \rightarrow \pi^+\pi^+\pi^-\bar{\nu}_e$. Can you try to explain what is going on? Hint: look up the quark contents of these particles; try to draw Feynman diagrams for these processes; remember that you can have a gluon which couples to a $q\bar{q}$ pair.

• Problem 2

Rapidity (y) is a kinematical variable which is useful for describing hadron-hadron collisions, e.g., at the Tevatron.

Consider $p\bar{p}$ or pp collisions. One could in principle describe the kinematics of produced particles in terms of cylindrical coordinates, i.e., by taking the z-axis to point along the proton direction, and using the momentum component in the XY plane (transverse momentum, P_T), the angle between the z-axis and momentum vector (θ) and the angle between the P_T vector and the X-axis (ϕ). It is however more convenient to replace the θ variable with rapidity, defined as

$$y = \frac{1}{2} \ln \frac{E+P_L}{E-P_L}$$

where E is the energy of the particle and P_L is the longitudinal momentum, i.e., the z -component of momentum. Let us now investigate some properties of the rapidity variable.

(i) We will show later in the quarter that the cross-section for particle production is proportional to the phase space factor d^3P/E , where P and E are the three-momentum and energy of the particle respectively. Show that

$$\frac{d^3P}{E} = \frac{1}{2} dP_T^2 dy d\phi$$

Since the cross-section for particle production depends mostly on P_T and very weakly on y , particle production is expected to be uniform in rapidity.

(ii) Show that under a longitudinal boost with velocity β the rapidity of a particle changes as

$$y \rightarrow y + \Delta y$$

with $\beta = \tanh \Delta y$. The significance of this is that although the LAB frame coincides with the center of mass of the $p\bar{p}$ or pp collisions, it does not necessarily coincide with the center of mass of the parton-parton collisions (a parton is a quark or an anti-quark, or a gluon inside the proton). This is because the partons carry only a fraction of the p and $\bar{p}$ momentum. Also, to first order, the partons carry no momentum perpendicular to the direction of motion of the proton or antiproton. Then, in $P_T - y - \phi$ space the transformation from LAB variables to parton-parton CM variables is very simple, since P_T and ϕ are invariant under a longitudinal boost. (Longitudinal here means along the beam axis).

(iii) Show that for a particle of mass m in the LAB frame

$$-\frac{1}{2} \ln \frac{s}{m^2} < y < \frac{1}{2} \ln \frac{s}{m^2}$$

where s is the square of the center of mass energy of the $p\bar{p}$ or pp collisions, and $m \ll \sqrt{s}$

(iv) Show that as $m \rightarrow 0$, or equivalently $m \ll E$

$$y \rightarrow \eta = \ln(\cot \frac{\theta}{2})$$

The quantity η is called *pseudorapidity*. Plot η as a function of the polar angle θ in degrees.

(v) Now consider a high transverse-momentum-transfer collision, such as $q\bar{q} \rightarrow q\bar{q}$. Of course the q and the $\bar{q}$ emerging from the collision are not seen directly in the detector. They will *fragment* into a number of hadrons ($\pi, K, p, \dots$). These hadrons have small transverse momentum with respect to the original direction of flight of the quark and anti-quark. As a result, they will be bunched closely around the original q and $\bar{q}$ directions, and what is seen in the detector are two sprays (*jets*) of particles. Let q_L and q_T be typical values for the momentum of the hadrons from the quark fragmentation as measured along and transverse from the direction of flight of the quark. Take $q_T \ll q_L$ and $m \ll q_L$, where m is the typical mass of these hadrons.

Show that jets are approximately circular in $\eta - \phi$ space, i.e. that $\Delta\eta \approx \Delta\phi$, where $\Delta\eta$ and $\Delta\phi$ are the typical spreads of jets in the η and ϕ coordinates. Also, show that the $\eta - \phi$ size of a jet from the fragmentation of a quark of a given transverse momentum is independent of the rapidity of the original quark.

For these reasons, jet momenta are often measured by summing up the momenta of all the hadrons within a cone in $\eta - \phi$ space. These algorithms are called *fixed cone* algorithms.

• Problem 3

Consider a parallel and infinitely narrow beam of particles incident at right angles upon a plate of some material. Assume that these particles are of the same kind, and have the same momentum. Neglect energy loss in the plate. Work in the small angle scattering approximation. The number of incident particles is N_0 .

Take a system of cartesian coordinates with the origin at the point of incidence, and the z -axis along the direction of motion. Let Θ_0^2 be the mean square angle of scatter **per unit length** in the plate in the xz or yz plane. Let $N(z, y, \theta_y) dy d\theta_y$ be the number of particles at depth z inside the plate having deflections between y and $y + dy$ and angle between θ_y and $\theta_y + d\theta_y$ in the yz plane.

(i) Show that

$$\frac{\partial N}{\partial z} = -\theta_y \frac{\partial N}{\partial y} + \frac{1}{2} \Theta_0^2 \frac{\partial^2 N}{\partial \theta_y^2}$$

Hint: Try to write down an expression for the change in $N(z, y, \theta_y)$ for $z \rightarrow z + dz$.

If you cannot obtain this differential equation, take it as a given and proceed with the rest of the problem.

(ii) Verify that a solution of the differential equation in (i) is

$$N(z, y, \theta_y) dy d\theta_y = \frac{\sqrt{3} N_0}{\pi} \frac{1}{\Theta_0^2 z^2} \exp\left[-\frac{2}{\Theta_0^2} \left(\frac{\theta_y^2}{z} - \frac{3y\theta_y}{z^2} + \frac{3y^2}{z^3}\right)\right]$$

(do not worry about the normalization for now). Note that the $y\theta_y$ term in the exponent implies that at a given depth in the plate the displacement (y) and the angle (θ_y) are correlated. This makes sense, e.g., if a particle has a positive displacement with respect to its original direction of motion, you should expect that on average its direction will also be such that θ_y is positive. We'll see this explicitly in part (v).

(iii) Integrate the result of (ii) over y and show that $N(z, \theta_y) d\theta_y$ is a properly normalized gaussian of mean 0 and standard deviation $\Theta_0 \sqrt{z}$.

(iv) Integrate the result of (ii) over θ_y and show that $N(z, y) dy$ is a gaussian of mean 0 and standard deviation $\frac{1}{\sqrt{3}} \Theta_0 z^{3/2}$. Here you should not worry about the normalization, since we have already seen in (iii) that the normalization is OK.

(v) Let t be the thickness of the plate. Show that the angular distribution of particles emerging from the plate with displacement $y = d$ is a gaussian, with mean different from zero. Find the mean and the standard deviation of the gaussian in terms of d , t , and Θ_0 . Verify that this standard deviation is smaller than the standard deviation calculated in (iii) for $z = t$.

• Problem 4

One of the most challenging experimental problems is that of distinguishing between different charged hadrons, e.g., proton, kaons and

pions. The momentum of the charged track is measured by measuring its deflection in a magnetic field. Once the momentum is measured, the particle species can be identified using some measurement that is a function of velocity (and therefore mass, since P is measured and $P = \gamma M\beta$).

One such possible measurement is that of dE/dX , which depends on β . The energy loss can be measured by measuring the size of the electronic pulse recorded in a gas chambers or a silicon detector. The charge corresponding to the electronic pulse is proportional to the number of electron-ion pairs produced as the track travels the detector, and this number is in turn proportional to dE/dX . In practice, many such measurements are needed to average out the large intrinsic fluctuations in the underlying physical process. This can be done in a drift chamber for a collider detector, where the charged track crosses many wire planes, and therefore several independent measurements of dE/dX can be averaged. To get a feeling for the separation between particle species that can be achieved on the basis of dE/dx , plot dE/dx in argon as a function of momentum for muons, pions, kaons, and protons. Argon, mixed with hydrocarbons, is a gas commonly used in drift chambers. Take the mean excitation energy in Ar to be $I = 16$ eV. You can neglect the density effect correction to the Bethe-Bloch formula, see the PDG. Plot momenta on a logarithmic scale between 100 MeV/c and 10 GeV/c; you can also plot dE/dX in arbitrary units if you wish. Note that a *good* dE/dX system can have resolution of order 6%.

Another possibility is to measure β by measuring the time taken by the track to travel a known distance (*time-of-flight*). In a collider detector, this distance could be the distance between the beamline and scintillation detectors placed at the exit of the drift chamber. Plot this time as a function of momentum between 500 MeV/c and 10 GeV/c for a drift chamber radius $R = 1.5$ m for muons, pions, kaons, and protons produced with no longitudinal momentum, i.e., with polar angle $\theta = 90^\circ$. (It is probably best if you plot it as a function of the log of the momentum rather than the momentum). First do this ignoring the effects of the bend in the magnetic field. Then make the same plots including the effects of an axial (i.e. in the z direction) magnetic field of 1.5 Tesla.

Reminder: the radius of curvature ρ for a charged particle in a constant

magnetic field is given approximately by $\rho = P_{\text{perp}}/(0.3B)$ where P_{perp} is the magnitude of the component of the particle momentum perpendicular to the magnetic field in GeV/c, B is the magnetic field in Tesla, and ρ is in meters.

It is very hard to achieve timing resolutions better than 100 psec in such systems.

• Problem 5

Identification of muons is another important issue in experimental high energy physics. In order to separate muons from charged hadrons (pions, kaons, protons, etc.) large amounts of absorber, e.g. iron, are placed in the path of the particles. The idea is to then detect the muons as they exist the material, while hadrons are likely to have stopped due to their hadronic interactions in the absorber.

Even in the absence of economic constraints, the absorber cannot be made infinitely thick, because then muons will also stop due to energy loss through dE/dx . Thus a given absorber thickness determines the minimum momentum for muon detection. Furthermore, for a given absorber thickness there is an associated probability that a hadron will go through the absorber without interacting.

This probability determines the best possible, i.e. the lowest, probability to misidentify a charged hadron as a muon. In reality the probability is higher because of several effects, e.g., (a) some of the debris from the hadronic shower can penetrate to the back of the absorber, (b) the pions from the hadronic shower can decay into muons and these muons can then penetrate to the back of the absorber, (c) if the incident hadron is a pion or a kaon, it has a finite probability of decaying into a muon ($\pi^+ \rightarrow \mu^+ \nu_\mu$ or $K^+ \rightarrow \mu^+ \nu_\mu$) before it interacts.

Determine the thickness of iron for a muon detector with momentum thresholds of 1, 5 and 10 GeV/c. Then find the corresponding best possible hadron misidentification probability.

Do not attempt precise calculation, e.g., by integrating the Bethe-Bloch equation. Simply look up in the PDG the minimum ionizing energy and the interaction length in iron and do a back-of-the-envelope calculation.